

# Quick Reference Guide: Standards for Mathematical Practice High School

To become college and career ready, students must be able to problem-solve, reason and prove, communicate, represent, make connections, conceptualize, and strategize in mathematics. While content standards are specified by grade level, the Standards for Mathematical Practice evolve and mature over the years along with students' cognitive development. By integrating content and practice standards, students become practitioners of the discipline of mathematics. **This guide describes how mathematically proficient high school students might demonstrate the eight Standards for Mathematical Practice in the [Massachusetts Curriculum Framework for Mathematics](#).**

“The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the habits of mind—used to create the results.”

-Al Cuoco, Paul Goldenberg, &  
June Mark

## 1. Make sense of problems and persevere in solving them.

Mathematically proficient high school students analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution.

**Example:** Students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.

## 2. Reason abstractly and quantitatively.

Mathematically proficient high school students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

**Example:** Students might mechanically manipulate a function to reveal zeros and other key features, then pause to consider the meaning of these features in the context of the relationship the function represents.

## 3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient high school students understand and use stated assumptions, definitions, and previously established results in constructing verbal and written arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.

**Example:** Students might write explanatory texts that convey their mathematical analyses and thinking, using relevant and sufficient facts, concrete details, quotations, and coherent development of ideas.

## 4. Model with mathematics.

Mathematically proficient high school students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that future revision may be needed. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas.

**Example:** A student might use geometry to solve a bridge design problem or use a function to describe how the number of engines assembled depends on the number of person-hours.

## 5. Use appropriate tools strategically.

Mathematically proficient high school students consider the available tools when solving a mathematical problem. Proficient students are sufficiently familiar with tools appropriate for high school to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from each and its limitations.

**Example:** Students might analyze graphs of functions and solutions generated using a graphing calculator.

## 6. Attend to precision.

Mathematically proficient high school students communicate precisely to others both verbally and in writing, adapting their communication to specific contexts, audiences, and purposes. They develop the habit of using precise language, not only as a mechanism for effective communication but also as a tool for understanding and solving problems.

**Example:** A student might describe the behavior of a function using key terms such as *intercepts*, *increasing*, *maximum*, and *line of symmetry*.

## 7. Look for and make use of structure.

Mathematically proficient high school students look closely to discern a pattern or structure. They can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

**Example:** Students might describe  $5 - 3(x - y)^2$  as 5 minus a positive number times a square, and use that description to realize that the expression's value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## 8. Look for and express regularity in repeated reasoning.

Mathematically proficient high school students notice if calculations are repeated, and they look for both general methods and shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details and continually evaluating the reasonableness of their intermediate results.

**Example:** Students might notice the regularity in the way terms sum to zero when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$ —leading them to the general formula for the sum of a geometric series.