Implications of the Common Core State Standards for Mathematics

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History of Development of CCSS

- July 2009: The development of the College and Career Ready Standards draft, outlining topic areas
- October 2009: Public release of the College and Career Ready Standards
- January 2010: Public release of Draft 1
- March 2010: Public release of Draft 2
- June 2, 2010: Final release of Common Core State Standards with approval of the Validation Committee
- (Note: These are NOT federal standards: they are a state-level coordinated effort led by National Governors Association-NGA and the Council of Chief State School Officers-CCSSO.)
Common Core State Standards represent an opportunity – once in a lifetime!

Structure of the Process

- Lead Writers
- Writing Teams
- State Review Teams
- Professional Organizations
- Validation Team
- State Adoption Process
States Adopting the CCSS (Yellow)

What adoption means

- A state adopts 100% of the common core K-12 standards in ELA and mathematics (word for word).

- With option of adding up to an additional 15% of standards on top of the core, but Michigan has chosen not to add any additional content.
Why Common Standards Now?

- Disparate standards across states
- Student mobility
- Global competition
- Today’s jobs require different skills

CCSSI 2010; www.corestandards.org

Why is This Important for Students, Teachers, and Parents?

- Prepares students with the knowledge and skills they need to succeed in college and work
- Ensures consistent expectations regardless of a student’s zip code
- Provides educators, parents, and students with clear, focused guideposts
Criteria for the Standards

- Fewer, clearer, and higher standards
- Aligned with college and work expectations
- Include rigorous content and application of knowledge through high-order skills
- Build upon strengths and lessons of current state standards
- Internationally benchmarked, so that all students are prepared to succeed in our global economy and society
- Based on evidence and research

CCSSI 2010; www.corestandards.org

As a member of the Validation Committee

- Reviewed and commented College and Career Readiness
- Reviewed and commented drafts of K-12 Standards four times
- Made a judgment regarding whether they met the criteria
Types of evidence used

• Data on ACT and SAT scores and performance in 1st year college courses
• Analysis of college syllabi and surveys
• Surveys with business members
• Examination of college level math and math-client fields
• Benchmarked to International Standards
• Evidence on student learning studies

Keep in mind....

“These Standards do not dictate curriculum or teaching methods. For example, just because Topic A appears before Topic B in a given grade, it does not mean that Topic A must be taught before Topic B.”

CCSS 2010, p. 5
Only a first step

Standards are essential, but inadequate. Along with standards,

- Educators must be given resources, tools, and time to adjust classroom practice.
- Instructional materials needed that align to the standards.
- Assessments must be developed to measure student progress.
- Federal, state, and district policies will need to be reexamined to ensure they support alignment of the common core state standards with student achievement.

CCSSI 2010; www.corestandards.org

• www.corestandards.org
Key Advances

Focus and coherence
• Focus on key topics at each grade level.
• Coherent progressions across grade levels.

Balance of concepts and skills
• Content standards require both conceptual understanding and procedural fluency.

Mathematical practices
• Foster reasoning and sense-making in mathematics.

College and career readiness
• Level is ambitious but achievable.

Building from past work

NCTM process standards:
• problem solving,
• reasoning and proof,
• communication,
• representation, and
• connections.
Five Strands of Mathematical Proficiency
(Adding It Up, NRC 2001)

• adaptive reasoning;
• strategic competence;
• conceptual understanding (comprehension of mathematical concepts, operations and relations);
• procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately); and
• productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

Eight Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
Eight Mathematical Practices

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Focus on Understanding

“The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word ‘understand’ are often especially good opportunities to connect the practices to the content.”

CCSS, 2010, p. 8
Standards, Curriculum, and Pedagogy

“These Standards do not dictate curriculum or teaching methods. For example, just because Topic A appears before Topic B in a given grade, it does not mean that Topic A must be taught before Topic B.”

CCSS 2010, p. 5

Understanding

• Avoids too much emphasis on procedure
• Facilitates flexibility
• Use of analogous problems
• Stronger representations
• Justified conclusions
• Application to practical situations
• Mindful use of technology
• Accurate and clear explanations
• Metacognition
Design and Organization

- **Content standards** define what students should understand and be able to do
- **Clusters** are groups of related standards
- **Domains** are larger groups that progress across grades

K-5 Domains

<table>
<thead>
<tr>
<th>Domains</th>
<th>Grade Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting and Cardinality</td>
<td>K only</td>
</tr>
<tr>
<td>Operations and Algebraic Thinking</td>
<td>1-5</td>
</tr>
<tr>
<td>Number and Operations in Base Ten</td>
<td>1-5</td>
</tr>
<tr>
<td>Number and Operations--Fractions</td>
<td>3-5</td>
</tr>
<tr>
<td>Measurement and Data</td>
<td>1-5</td>
</tr>
<tr>
<td>Geometry</td>
<td>1-5</td>
</tr>
</tbody>
</table>
Middle Grades Domains

<table>
<thead>
<tr>
<th>Domains</th>
<th>Grade Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio and Proportional Relationships</td>
<td>6-7</td>
</tr>
<tr>
<td>The Number System</td>
<td>6-8</td>
</tr>
<tr>
<td>Expressions and Equations</td>
<td>6-8</td>
</tr>
<tr>
<td>Functions</td>
<td>8</td>
</tr>
<tr>
<td>Geometry</td>
<td>6-8</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>6-8</td>
</tr>
</tbody>
</table>

High School Conceptual Categories and Domains

- **NUMBER AND QUANTITY**
  - The Real Number System
  - Quantities
  - The Complex Number System
  - Vector and Matrix Quantities
- **ALGEBRA**
  - Seeing Structure in Expressions
  - Arithmetic with Polynomials and Rational Expressions
  - Creating Equations
  - Reasoning with Equations and Inequalities
High School Conceptual Categories and Domains

• FUNCTIONS OVERVIEW
  – Interpreting Functions
  – Building Functions
  – Linear, Quadratic and Exponential Models
  – Trigonometric Functions

• MODELING

High School Conceptual Categories and Domains

• GEOMETRY
  – Congruence
  – Similarity, Right Triangles and Trigonometry
  – Circles
  – Expressing Geometric Properties with Equations
  – Geometric Measurement and Dimension
  – Modeling with Geometry

• STATISTICS AND PROBABILITY
  – Interpreting Categorical and Quantitative Data
  – Making Inferences and Justifying Conclusions
  – Conditional Probability and the Rules of Probability
  – Using Probability to Make Decisions
Major Shifts K-5

- Numeration and operation intensified, and introduced earlier
  - Early place value foundations in grade K
  - Regrouping as composing / decomposing in grade
  - Decimals to hundredths in grade 4
- All three types of measurement simultaneously
  - Non-standard, English and Metric
- Emphasis on fractions as numbers
- Emphasis on number line as visualization / structure

How is there less

- Backed off of algebraic patterns K-5
- Backed off stats and probability in K-5
- Delayed content like percent and ratio and proportion
Major Shifts 6-8

- Ratio and Proportion focused on in grade 6
  - Ratio, unit rates, converting measurement, tables of values, graphing, missing value problems
- Percents introduced grade 6
- Statistics is introduced grade 6
  - Statistical variability (measures of central tendency, distributions, interquartile range, mean and absolute deviation, data shape)
- Rational numbers in grade 7
- One-third of algebra for all students in grade 8

Less in Middle Grades

- The Common Core Standards are not less in middle grades and will only be fewer if what happens in elementary leads to more students knowing the content and avoiding repetition.
Major Shifts 9-12

• Supports both/either continuing an integrated approach or a siloed approach (Algebra I, Geometry, Algebra II) or new models that synthesize the two.
• All students must master topics traditionally from algebra II or beyond
  – simple periodic functions
  – polynomials,
  – Radicals
  – More probability and statistics (correlation, random processes)
  – introduced to mathematical modeling

Now We can Focus on Instruction

[Diagram showing the cycle of professional development, intended curriculum, implemented curriculum, learning trajectories, instructional practices, high stakes assessment, instructional guidance and improvement, data, formative assessment, summative assessment.]
The Challenge

- Engagement
- Consider: What would happen if students were not required to take mathematics, could we attract them to the approaches we are using?
- Organize instruction in order to attract and engage students (do not interpret as lowering the standards)
- How to avoid an “algorithmic” or “procedural” interpretation?
- Attention to variation and recovery
- Equity of opportunity as we move towards increasing rigor
- How to address the accelerated 8th grade math issue?

Build to support learning trajectories

Developing “sequenced obstacles and challenges for students ... absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.”

Confrey 2007, quoted in CCSS 2010, p.5
Build to support learning trajectories

“One promise of common state standards is that over time, they will allow research on learning progressions to inform and improve the design of Standards to a much greater extent than is possible today.”

CCSS 2010, p.5

A learning trajectory/progression is:

...a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time

(Confrey et al., 2009)
Value of Learning Trajectories to Teachers

• Know what to expect about students’ preparation
• More readily manage the range of preparation of students in your class
• Know what teachers in the next grade expect of your students.
• Identify clusters of related concepts at grade level
• Clarity about the student thinking and discourse to focus on conceptual development
• Engage in rich uses of classroom assessment

Example

• Follow the development of *place value* in the Common Core Standards
Example: Place Value and Base Ten

Number and Operations in Base Ten

Kindergarten (K.NBT)

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., \(18 = 10 + 8\)); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Example One: Place Value and Base Ten

Number and Operations in Base Ten

1st grade (1.NBT)

Extend the counting sequence

(standards 1.NBT.1)

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones.

Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

(1.NBT.2)

3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

(1.NBT.3)

Use place value understanding and properties of operations to add and subtract.

(standards 1.NBT.4 through 1.NBT. 6)
Example One: Place Value and Base Ten

Number and Operations in Base Ten 2nd grade (2.NBT)

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (2.NBT.1)

2. Count within 1000; skip-count by 5s, 10s, and 100s. (2.NBT.2)

3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (2.NBT.3)

4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (2.NBT.4)

Use place value understanding and properties of operations to add and subtract.

(standards 2.NBT.5 through 2.NBT.9)

Observations about Place Value and Base Ten in Early Grades

- Grade K:
  - Foundation in bundling
  - Emphasis on the teen numbers

- Grade 1:
  - extends to 10, 20, 30, ...
  - Learn to compare

- Grade 2:
  - extend to 100 as a bundle of ten 10s
  - Extend to 100, 200, 300, ...
  - Expanded notation and comparison
Example One: Place Value and Base Ten

**Number and Operations in Base Ten**  
3rd grade  
(3.NBT)  
Use place value understanding and properties of operations to perform multi-digit arithmetic.  
(Round whole numbers to 100, add and subtract to 1000, and multiply multiples of 10 by one-digit numbers) (3.NBT.1 through 3.NBT.3)

**Number and Operations in Base Ten**  
4th grade  
(4.NBT)  
Use place value understanding and properties of operations to perform multi-digit arithmetic.  
(Generalize each digit is ten times the next, compare large numbers, round, add and subtract wholes, multiply two-digit by two-digit and one-digit by four-digit numbers.) (4.NBT.1-5)

**Number and Operations—Fractions**  
4th grade  
(4.NF)  
Understand decimal notation for fractions and compare decimal fractions.  
(Fractions with denominator 10 or 100 as decimals.) (4.NF.5 through 4.NF.7)

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Example One: Place Value and Base Ten

**Number and Operations in Base Ten**  
5th grade  
(5.NBT)  
Understand the place value system.  
(Generalize each digit to right of another is one-tenth the previous, patterns of zeros, multiplication and division, compare decimals to thousandths, round.) (5.NBT.1 to 5.NBT.4)

**Expressions and Equations**  
8th grade  
(8.EE)  
Work with radicals and integer exponents.  
(Express large and small numbers in scientific notation, and perform operations.)
## Learning Trajectories View of the Common Core Standards

<table>
<thead>
<tr>
<th>Content Strand</th>
<th>Kindergarten</th>
<th>Grade 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Place Value and Decimals</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compare, read, and write numbers from 0 to 20 (K.NBT.1)</td>
<td>1.NBT.2</td>
<td>1.NBT.2</td>
</tr>
<tr>
<td>Compare two single-digit numbers based on the meanings of the tens and ones digits (1.NBT.3)</td>
<td>1.NBT.3</td>
<td>1.NBT.3</td>
</tr>
<tr>
<td>Compare two two-digit numbers based on the meanings of the tens and ones digits (1.NBT.4)</td>
<td>1.NBT.4</td>
<td>1.NBT.4</td>
</tr>
</tbody>
</table>

### Learning Trajectories Display of the Common Core Standards

- Available late August as three posters: K-5, 6-8 and 9-12.
So What?

• If Common Core is successful, what will change?

For some states, higher standards
For some states, there will be no change in expected outcomes

Except: MORE STUDENTS SHOULD BE ABLE TO SUCCEED DUE TO COORDINATION, ECONOMIES OF SCALE AND BETTER FOCUS.

As a living document, the Standards should be adjusted over time as a result of evidence

Next steps

• Consider a phasing in model
• Launch professional development efforts on a focus on the practices
• Work on strategies for the transition points (5-6) and (8-9)
• Review the ELA standards for scientific and reading/writing in science and technical areas
• Implement changes at grade level but concentrate on the trajectories
Next steps

• Develop new summative assessments (federal program supporting state coalitions due 2014)
• Develop effective formative assessment and diagnostic approaches around the learning trajectories strengthening discourse and instructional guidance
• Really need the younger generation of teachers to pick up this ball and carry it.
• Talk to Dan Ladue and Ruth Ann Hodges

“It is time to recognize that Standards are not just promises to our children, but promises we intend to keep.” CCSS 2010, p. 5
• Questions?
Learning Trajectories at High School

• Use levels of difficulty to describe the progressions

• Use domains or strands to articulate the clusters of topics

• Follow the development of concept of Functions in the Common Core Standards
The Development of Functions: Grade 6

No domain for functions; only equations and expressions, and ratio and proportion

Apply and extend previous understandings of arithmetic to algebraic expressions.
2. Write, read, and evaluate expressions in which letters stand for numbers.
c. Evaluate expressions of specific values of their variables. Include expressions that arise form formula used in real-world problems...”

Apply and extend previous understandings of arithmetic to algebraic expressions.
9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Understand ratio concepts and use ratio reasoning to solve problems.
a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Unpacking Required

• What are the relationships among expressions, relations, formula, equations and functions? What about variables and quantities?
• How do students interpret these?
• At what point should the explicit idea of function be introduced?
Development of Functions: Grade 7

No domain for functions; only equations and expressions and ratio and proportion

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers...
      (7. EE.4a)
   b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers...
      (7. EE.4b)

Analyze proportional relationships and use them to solve real-world and mathematical problems

2. Recognize and represent proportional relationships between quantities. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Test proportionality using table or graph, identify constant of proportionality, represent as an equation, link to unit rates.

(7. RP.2a – d)

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Development of Functions: Grade 8

Has the domain “Functions”

Define, evaluate, and compare functions.

1. (Definition) (8.F.1)
2. Compare properties of two functions each represented in a different way (8.F.2)
3. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (8.F.3)

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function... (8.F.4)
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph...
   Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (8.F.5)

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpret unit rate as slope... (8.EE.5)
6. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function... (8.EE.6)

Analyze and solve linear equations and pairs of simultaneous linear equations. (8.EE.7; 8.EE.8)

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Observations

- Before high school, there is a considerable foundation for functions, including multiple representations
- The language is delayed
- Sequences are not used to introduce (?: no pattern work)
- Relationship of functions to equations, expressions, and formulas is not clarified
- The tension between “variable as missing value” and “variable as quantity that varies” is not addressed explicitly
- Based in ratio and proportion only
Advantages of the high school learning trajectories display

• Supports the use of siloed or integrated curricula, or some other approach

• Can be coordinated with assessments, by identifying clearly what will be assessed, when and how