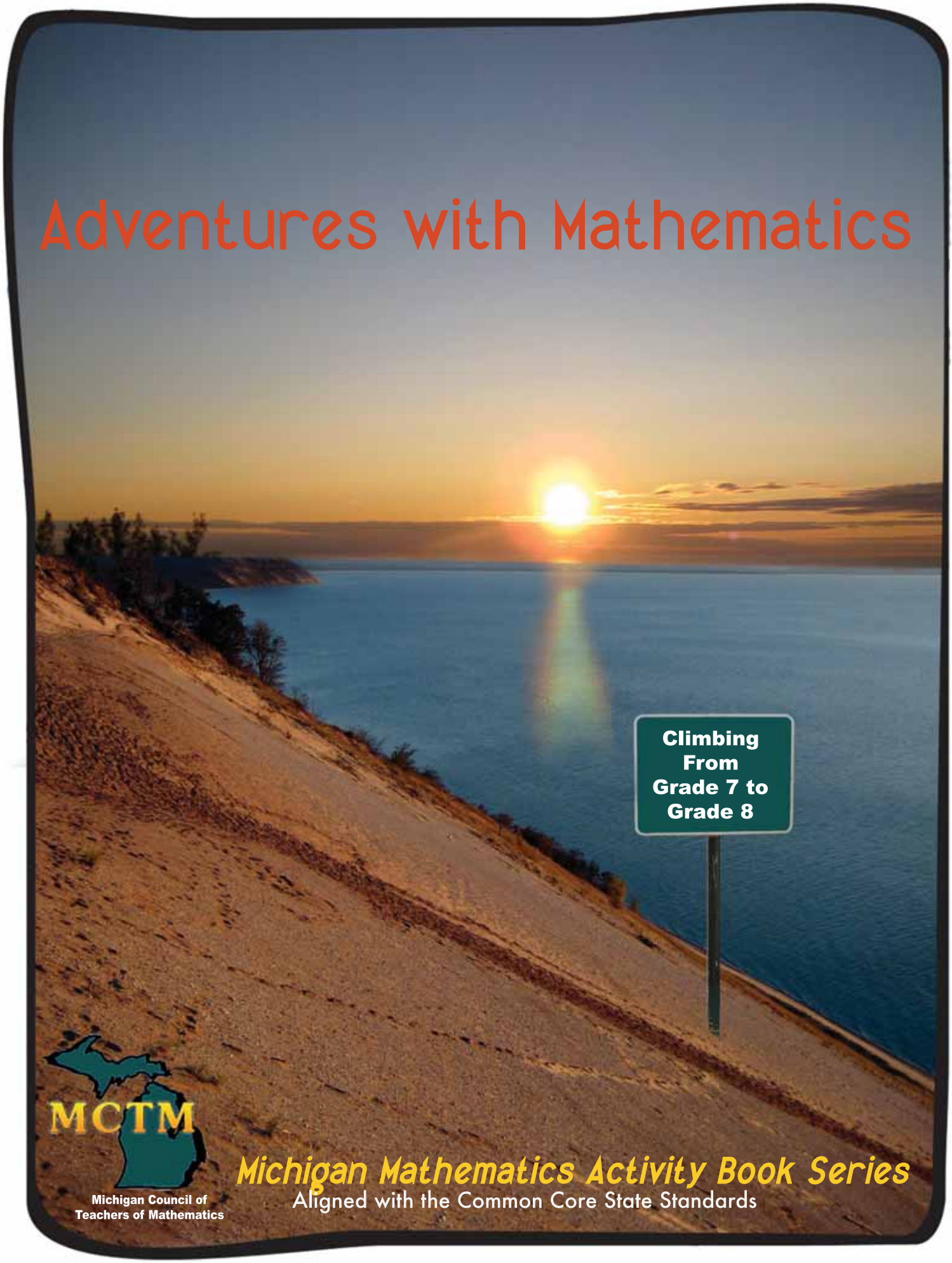
Adventures with Mathematics



MCTM

Michigan Council of
Teachers of Mathematics

Michigan Mathematics Activity Book Series

Aligned with the Common Core State Standards

Adventures with Mathematics: Climbing from Grade 7 to Grade 8

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MCTM would like to acknowledge all who have contributed to this series of books. Your work is greatly appreciated.

Proceeds benefit the Michigan Council of Teachers of Mathematics Scholarship Endowment Fund.

Michigan Mathematics Activity Book Series

Supported By:

Grand Valley State University
Eastern Michigan University
Michigan State University
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Siena Heights University 1919
CMU Central Michigan University
University of Michigan Dearborn
cptm center for proficiency in teaching mathematics
Michigan Department of Education

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GRADE 7 COMMON CORE STATE STANDARDS ALIGNMENT OF ACTIVITIES AND SHORT PROBLEMS

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CC.7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.			X	X			X	X				X	X	X						X			20, 27, 28, 31, 42, 43, 44, 45, 48, 55
CC.7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.			X				X	X					X										28, 31, 39, 48, 55
CC.7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.																	X						
CC.7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.																				X			32
CC.7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.																							
CC.7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.							X							X				X		X			20, 41
CC.7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.																X							1, 30, 37, 62
CC.7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."						X										X							38, 58
CC.7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations as strategies to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.				X		X		X	X	X	X	X	X	X	X					X	X		1, 2, 3, 9, 10, 11, 17, 18, 19, 24, 25, 30, 34, 37, 41, 58, 62
CC.7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	X		X	X						X				X					X	X	X		1, 11, 18, 19, 24, 35, 36, 49, 51, 59, 60, 62
CC.7.EE.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.																							24, 36, 59
CC.7.NS.1a Describe situations in which opposite quantities combine to make 0.																							30, 37, 62
CC.7.NS.1b Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.						X																	30, 37, 62

CC.7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.																						15, 30
CC.7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.					X		X							X								1, 10
CC.7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.														X				X				30, 37, 62
CC.7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.										X				X								30, 37, 62
CC.7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.					X					X				X				X				1, 2, 3, 4, 6, 7, 10, 13, 23, 37, 53, 62
CC.7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.																						
CC.7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.			X		X		X	X		X	X	X	X					X	X			1, 17, 18, 23, 25, 30, 34, 37, 39, 52, 56, 58, 62
CC.7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	X		X	X			X	X	X	X	X	X	X	X				X				2, 4, 14, 19, 26, 32, 34, 39, 52, 53
CC.7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	X			X							X			X					X			12, 16, 33, 54
CC.7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	X		X	X			X		X		X		X	X				X	X			2, 5, 8, 11, 12, 14, 16, 19, 32, 39, 50, 54
CC.7.RP.2c Represent proportional relationships by equations.	X		X	X			X		X		X		X	X				X	X			11, 19, 50, 54, 60
CC.7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	X		X	X				X	X	X				X					X			11, 36, 38, 59
CC.7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.			X		X			X	X	X	X	X							X	X			16, 36, 38, 39, 40, 58
CC.7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.																					X	X	29
CC.7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.			X																			X	29, 56, 57
CC.7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.																					X		46, 47
CC.7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.																					X		46, 47, 56, 57
CC.7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.																						X	61
CC.7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.																						X	29
CC.7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.																						X	
CC.7.SP.7b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.																						X	
CC.7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.																						X	61
CC.7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.																						X	
CC.7.SP.8c Design and use a simulation to generate frequencies for compound events.																						X	

G = Geometry, EE = Expressions and Equations, NS = The Number System, RP = Ratios and Proportions, SP = Statistics and Probability

Beautiful Bracelets

Strands:

Number & Operations	
Algebra	X
Measurement	X
Geometry	
Data & Probability	

Materials:

- Three colors of embroidery floss or friendship bracelet string
- Scissors
- Ruler or measuring tape
- Graph paper (optional)

Where?

Outside	
Inside	X
On-line	
On-site	

Have you ever wondered how to adjust directions for a project to make things exactly the size you need instead of one-size-fits-all? In this activity you will make friendship bracelets, gather data, and analyze the result. In the end, you will be able to predict the length of string needed to make a bracelet just the right size.

Making a Friendship Bracelet:

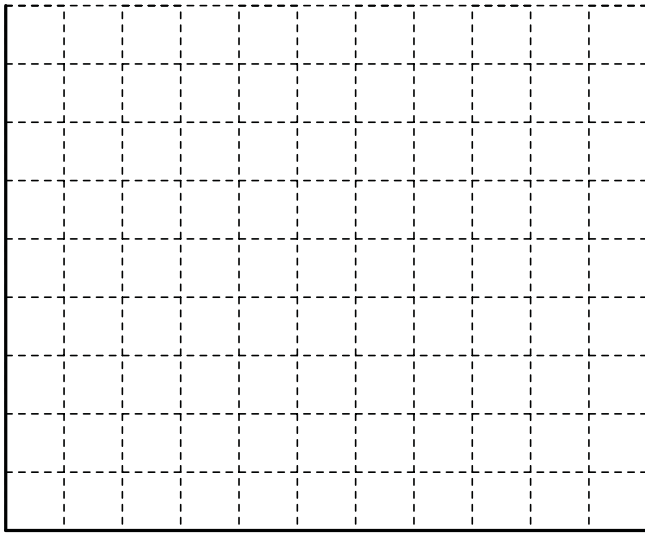
1. Choose three colors of string. Cut one 36-inch string of each color.
2. Put the three strings together, aligning the ends.
3. Fold the strings in half.
4. Hold on to one end of the strings and twist at the other end of the strings.
5. Once the string is tightly twisted and there is no buckling, fold the bracelet in half once again. Let the bracelet twist around itself. It will do this automatically.
6. Tie a knot less than one inch from the end of the bracelet to hold all the loose ends together.

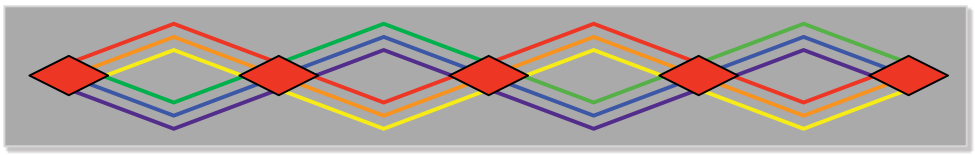
Analyzing the Results:

7. In the table below, record the full length of the finished bracelet from end to end without stretching it.
8. Complete the table for the string lengths listed below.
   Repeat steps 1 through 7 to make each bracelet.

String length, *S*	Length of bracelet, *B*
30 inches	
36 inches	
41 inches	
48 inches	

9. Graph your data points on a coordinate grid. Label the axes. Let the string length be the independent variable (horizontal axis) and bracelet length the dependent variable (vertical axis).
10. What does the graph tell you about the relationship between the length of the string and the length of the bracelet? Describe the relationship.
11. What does the slope (steepness) of the graph represent in terms of bracelets and string lengths? How do you know?
12. How much string is needed for a person who wants to have a finished bracelet that is 8 inches long?
13. How much string is needed if someone wants to make a 12-inch necklace using the same technique?
14. How many bracelets that are 7 inches long can you make with 96 inches of each color of string?
15. If you had 12 yards of embroidery floss for each of the three colors, how many 7-inch bracelets could you make?





Extensions:

16. Find an equation that allows you to predict:
    a. The length of string, S, (of each of 3 colors) you need to make one bracelet that is B inches long
    b. The number of bracelets, N, you can make that are B inches long from 12 yards of embroidery floss.
17. How do the equations in problems 16a and 16b relate to each other?
18. Repeat this experiment for braided bracelets. You can find instructions for braiding on the Internet.

Go, Go, Van Gogh!

Strands:

Number & Operations	
Algebra	
Measurement	X
Geometry	X
Data & Probability	

Materials:

- *Go, Go, Van Gogh* playing cards
- Ruler
- Compass
- Protractor
- Angle ruler (find free template online at http://www.teachervision.fen.com/tv/printables/scottforesman/Math_6_TTT_6.pdf)

Where?

Outside	
Inside	X
On-line	
On-site	

They say a picture says a thousand words. In this game, players find the true meaning of this famous saying.

Set-Up:

- Cut out and shuffle the playing cards on the next page.
- Decide who will be the Illustrator and who will be the Describer.

Describer:

1. Draw a card from the top of the stack.
2. Without showing the image to the Illustrator or saying the name of the image, describe to the Illustrator how to draw the picture using mathematical vocabulary. The Illustrator may use a pencil, ruler, compass, protractor, and angle ruler.
3. The words below can be used to help describe the image on the card.

Similar	Acute	Right
Perpendicular	Obtuse	Scalene
Bisector	Parallel	Isosceles
Equilateral	Polygon	Perimeter
Triangle	Hexagon	Area
Square	Angle	Prism
Rectangle	Segment	Vertex
Parallelogram	Congruent	Midpoint

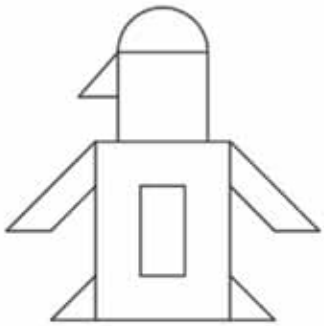
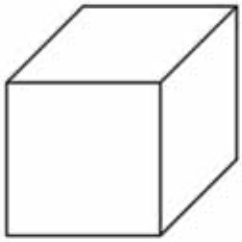
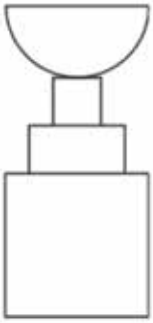
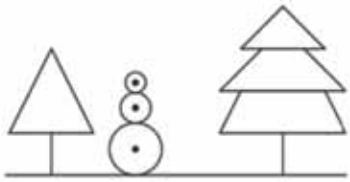
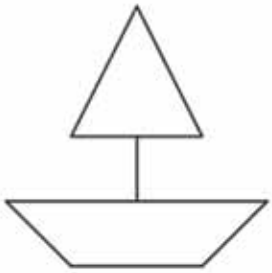
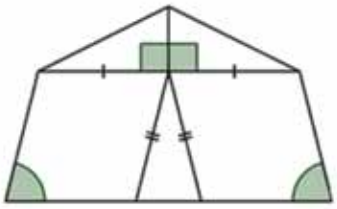
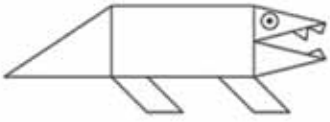
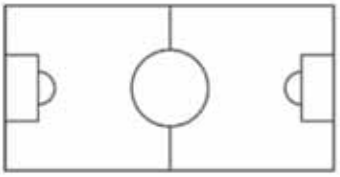
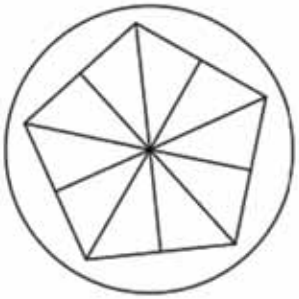
4. The Illustrator's job is to draw or construct the image with as much accuracy as possible. The new image can be larger or smaller than the image on the card, but it must be in the same proportion (similar).
5. Do not interrupt the work of the Illustrator even if the Illustrator is making a mistake.
6. Switch roles and play again!

Variations:

7. Create your own playing cards and add them to the deck.
8. The Illustrator's drawing is hidden until it is finished. Players can sit back-to-back so that the Describer cannot see the Illustrator's work.

Extensions

9. As Describer, think about how you described the drawings that the Illustrator was able to create most accurately. What did you say that made it easier for the Illustrator to draw an accurate image?
10. As Illustrator, what descriptions helped you draw the image most accurately? To what did you need to pay attention in order to draw an accurate image?

 House	 Penguin	 Cube
 The Stanley Cup	 Winter Wonderland	 Sailboat
 Tent	 Birdhouse	 Message
 Alligator	 Soccer Field	 Pizza

Go, Go,
Van Gogh!

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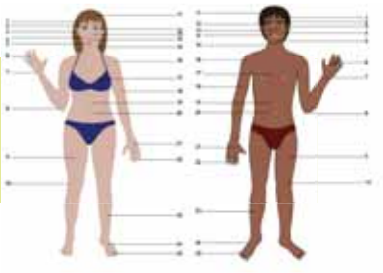
Human Proportions



Strands:

Number & Operations	
Algebra	X
Measurement	X
Geometry	
Data & Probability	X

Materials:

- Tape measure
- Paper for making a table
- Graph paper (optional)

Where?

Outside	
Inside	X
On-line	
On-site	

The human body has marveled scientists and mathematicians for centuries. That's right, mathematicians. They are particularly interested in the many interesting relationships among human dimensions. In this activity we will look at one of these relationships.

1. To begin, collect some data. Find at least five friends, (the more the better). Measure each person's height and forearm length. For this activity consider forearm length to be the length from the elbow (when it is resting on a table) to the tip of the middle finger. Record the data in a table so that you can tell which forearm length goes with which height.
2. From your table construct a scatter plot of the points with the horizontal (independent variable) axis representing the length of the forearm and the vertical (dependent variable) axis representing the friend's height.
3. Through these points draw a straight line that comes as close as possible to as many points as possible.

Use your line to answer the following questions:

4. Yao Ming is the tallest player in the National Basketball Association. His forearm measures an amazing 15.5 inches in length. What would you estimate Yao's height to be?

Extension:

5. a. Find the equation of your line from problem 3. (Remember all you need is two points to determine a unique line.)
   b. Use this equation to calculate the answer to problem 4. Compare your graph and equation answers. Why might they be different?
6. The Statue of Liberty in New York is 111 feet and 1 inch tall. If Lady Liberty is in the same proportion as the average human in your data, what could be the length of Lady Liberty's forearm?
7. A different proportion that has interested mathematicians is the relationship between wingspan (the length from the tip of the middle finger to the tip of the other middle finger with arms outstretched) and height. Try this activity using those measurements.